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NUMERICAL INVESTIGATION OF THE INTERACTION OF A PLANE WAVE WITH A MULTILAYERED CYLINDER IN THE GROUND

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We consider the two-dimensional nonstationary interaction problem of an intense compressional plane wave with an infinitely long multilayered deformable cylinder in the ground with account of elastic and plastic deformations. The rheology of the medium and of the cylinder materials is described by the equations of deformation theory [1] of elastoplastic bodies. In this case one uses as ground distortion function the generalized experimental dependence $\sigma_i = \sigma_i(\varepsilon, \varepsilon_i)$ (ε, ε_i, σ, σ_i are the first and second invariants of the deformation and stress tensors), taking into account the effect of bulk deformation on the nature of plasticity conditions [2] $\sigma_i = \sigma_i(\varepsilon_i)$, and simultaneously with the ground compression diagram $\sigma = \sigma(\varepsilon)$ satisfying in this specific case the sufficient conditions of uniqueness theorems and of minimum work of internal forces, obtained in [3] for a nonlinear medium. A numerical solution of the problem for small and finite deformations of the system investigated is implemented by a difference method of the crossing type [4] in Lagrangian variables without explicit separation of surface discontinuities. The approach mentioned has been used for numerical solution of two-dimensional collision problems of axially symmetric bodies with various obstacles, such as in [5-7].

In the present study specific numerical calculations of the problem are carried out for the case of streamlining of a wave of given intensity around two-layered and three-layered cylinders in the ground with account of wave diffraction by the external surface of the cylinder and of the nonlinearity (including linearity) of its deformable material. We investigate the effects of inelastic properties of the ground, physicomechanical characteristics, and the thickness of cylinder layers on the distributions of kinematic parameters and stresses in them. A comparison is carried out with the stress states of an elastic medium, generated during wave diffraction by a cylindrical cavity. We note that problems related to diffraction of elastic waves by cavities, solids in the presence of elastic fillers, and shells of various shapes in an unbounded elastic or acoustic medium, were treated in [8-12].

The present study is an extension of [13, 14] in the study of characteristic features of plane wave interactions with a multilayered cylinder in the ground and the behavior of its parameters under strong action.

Let the front of the intense plane wave propagating in the ground at the moment of time $t = 0$ be adjacent to the external surface of a long two- or three-layered cylinder in the ground. For a given wave intensity it is necessary to determine for $t > 0$ the stress-deformation states and the kinematic parameters of the ground and of the cylinder with account of wave diffraction, the cylinder material deformability, and the elastoplastic deformations generated in this case.

Since the problem is solved within the two-dimensional statement, the equations of motion of the ground and of the ring-shaped element of the cylinder are in the Lagrange variables (r, φ)

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$$\ddot{u}_r = \frac{1}{\rho_0 r}\left\{\frac{\partial}{\partial r}\left[r\sigma_{rr}\left(1+\frac{\partial u_r}{\partial r}\right)+\sigma_{r\varphi}\left(\frac{\partial u_r}{\partial\varphi}-u_\varphi\right)\right]+\frac{\partial}{\partial\varphi}\left[\sigma_{r\varphi}\left(1+\frac{\partial u_r}{\partial r}\right)+\right.\right.$$
$$\left.\left.+\sigma_{\varphi\varphi}\left(\frac{1}{r}\frac{\partial u_r}{\partial\varphi}-\frac{u_\varphi}{r}\right)\right]-\left[\sigma_{r\varphi}\frac{\partial u_\varphi}{\partial r}+\sigma_{\varphi\varphi}\left(1+\frac{\partial u_\varphi}{\partial\varphi}+\frac{u_r}{r}\right)\right]\right\},$$
$$\ddot{u}_\varphi = \frac{1}{\rho_0 r}\left\{\frac{\partial}{\partial r}\left[r\sigma_{rr}\frac{\partial u_\varphi}{\partial r}+\sigma_{r\varphi}\left(r+\frac{\partial u_\varphi}{\partial\varphi}+u_r\right)\right]+\right. \tag{1}$$
$$+\frac{\partial}{\partial\varphi}\left[\sigma_{r\varphi}\frac{\partial u_\varphi}{\partial r}+\sigma_{\varphi\varphi}\left(1+\frac{\partial u_\varphi}{\partial\varphi}+\frac{u_r}{r}\right)\right]+$$
$$\left.+\left[\sigma_{r\varphi}\left(1+\frac{\partial u_r}{\partial r}\right)+\sigma_{\varphi\varphi}\left(\frac{1}{r}\frac{\partial u_r}{\partial\varphi}-\frac{u_\varphi}{r}\right)\right]\right\},$$

where ρ_0 is the initial density, and u_r, u_φ are the radial and angular displacements of the medium.

According to deformation theory [1], the dependences between the stress and deformation components are written for all three media in the form

$$\sigma_{rr} = \lambda\varepsilon + 2G\varepsilon_{rr},\quad \sigma_{\varphi\varphi} = \lambda\varepsilon + 2G\varepsilon_{\varphi\varphi},\quad \sigma_{r\varphi} = G\varepsilon_{r\varphi}, \tag{2}$$
$$\lambda = \sigma/\varepsilon - (2/9)\sigma_i/\varepsilon_i,\quad G = (1/3)\sigma_i/\varepsilon_i$$

($\sigma(\varepsilon)$ $\sigma_i(\varepsilon_i)$ or $\sigma_i(\varepsilon, \varepsilon_i)$ are experimentally determined functions).

In the case of finite deformations we find for ε_{rr}, $\varepsilon_{\varphi\varphi}$ and $\varepsilon_{r\varphi}$ ($\varepsilon = \varepsilon_{rr} + \varepsilon_{\varphi\varphi}$)

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}+\frac{1}{2}\left[\left(\frac{\partial u_r}{\partial r}\right)^2+\left(\frac{\partial u_\varphi}{\partial r}\right)^2\right],$$
$$\varepsilon_{\varphi\varphi} = \frac{1}{r}\frac{\partial u_r}{\partial\varphi}+\frac{u_r}{r}+\frac{1}{2}\left[\left(\frac{1}{r}\frac{\partial u_r}{\partial\varphi}-\frac{u_\varphi}{r}\right)^2+\left(\frac{1}{r}\frac{\partial u_\varphi}{\partial\varphi}+\frac{u_r}{r}\right)^2\right], \tag{3}$$
$$\varepsilon_{r\varphi} = \frac{\partial u_\varphi}{\partial r}+\left(\frac{1}{r}\frac{\partial u_r}{\partial\varphi}-\frac{u_\varphi}{r}\right)+\frac{\partial u_r}{\partial r}\left(\frac{1}{r}\frac{\partial u_r}{\partial\varphi}-\frac{u_\varphi}{r}\right)+\frac{\partial u_\varphi}{\partial r}\left(\frac{1}{r}\frac{\partial u_\varphi}{\partial\varphi}+\frac{u_r}{r}\right).$$

In carrying out the calculations the experimental ground curves $\sigma(\varepsilon)$ and $\sigma_i(\varepsilon, \varepsilon_i)$, obtained in [15] for microgranular sand with initial density ρ_0 = 164 kg·sec²/m⁴, were approximated by the following dependences: for loading

$$\sigma(\varepsilon) = 9{,}951(\varepsilon/0{,}01)^2 + 9{,}604(\varepsilon/0{,}01)\ \text{for}\ 0 \leqslant \varepsilon \leqslant 10^{-2},$$
$$\sigma(\varepsilon) = 1{,}6835(\varepsilon/0{,}01)^2 + 26{,}1384(\varepsilon/0{,}01) - 8{,}2672\ \text{for}\ \varepsilon \geqslant 10^{-2}; \tag{4}$$

$$\sigma_i(\varepsilon, \varepsilon_i) = \sigma_i^{\text{н}}(\varepsilon_i) + [\sigma(\varepsilon) - 25]\left[\sigma_i^{\text{в}}(\varepsilon_i) - \sigma_i^{\text{н}}(\varepsilon_i)\right]/15,$$
$$\sigma_i^{\text{в}}(\varepsilon_i) = -14{,}997(\varepsilon_i/0{,}01)^2 + 35{,}037(\varepsilon_i/0{,}01),$$
$$\sigma_i^{\text{н}}(\varepsilon_i) = -10{,}218(\varepsilon_i/0{,}01)^2 + 23{,}035(\varepsilon_i/0{,}01)\quad \text{for}\ 0 \leqslant \varepsilon \leqslant 10^{-2},$$
$$\sigma_i^{\text{в}}(\varepsilon_i) = 20{,}04\left(\frac{\varepsilon_i - 0{,}03}{0{,}02}\right)\left(\frac{\varepsilon_i - 0{,}04}{0{,}03}\right) - 27{,}18\left(\frac{\varepsilon_i - 0{,}01}{0{,}02}\right)\times$$
$$\times\left(\frac{\varepsilon_i - 0{,}04}{0{,}01}\right) + 28{,}54\left(\frac{\varepsilon_i - 0{,}01}{0{,}03}\right)\left(\frac{\varepsilon_i - 0{,}03}{0{,}01}\right),$$
$$\sigma_i^{\text{н}}(\varepsilon_i) = 12{,}82\left(\frac{\varepsilon_i - 0{,}03}{0{,}02}\right)\left(\frac{\varepsilon_i - 0{,}04}{0{,}03}\right) - 17{,}23\left(\frac{\varepsilon_i - 0{,}01}{0{,}02}\right)\left(\frac{\varepsilon_i - 0{,}04}{0{,}01}\right) + \tag{5}$$
$$+ 18{,}84\left(\frac{\varepsilon_i - 0{,}01}{0{,}03}\right)\left(\frac{\varepsilon_i - 0{,}03}{0{,}01}\right)\quad \text{for}\ 10^{-2} < \varepsilon_i \leqslant 3{,}27\cdot 10^{-2},$$
$$\sigma_i^{\text{в}}(\varepsilon_i) = 27{,}688656 + 170{,}2108(\varepsilon_i - 3{,}27\cdot 10^{-2}),$$
$$\sigma_i^{\text{н}}(\varepsilon_i) = 17{,}7001 + 170{,}2108(\varepsilon_i - 3{,}27\cdot 10^{-2});$$

for unloading

$$\sigma(\varepsilon) = a\varepsilon^2 + b\varepsilon + c\quad \text{for}\ \frac{1}{2}(\varepsilon^* + \varepsilon_0) < \varepsilon < \varepsilon_0,$$

$$\sigma(\varepsilon) = d(\varepsilon - \varepsilon^*)^2\quad \text{for}\ \varepsilon^* \leqslant \varepsilon \leqslant \frac{1}{2}(\varepsilon^* + \varepsilon_0),$$

$$a=\frac{\sigma(\varepsilon_0)-(\varepsilon_0-0{,}5\varepsilon_1-0{,}5\varepsilon^*)\sigma'(\varepsilon_0)}{(\varepsilon_0-\varepsilon^*)(\varepsilon_1-\varepsilon_0)},\quad b=\sigma'(\varepsilon_0)-2a\varepsilon_0,$$

$$d=(a\varepsilon_1+b/2)/(\varepsilon_1-\varepsilon^*),\quad c=d(\varepsilon_1-\varepsilon^*)^2-a\varepsilon_1^2-b\varepsilon_1,$$
$$\varepsilon_1=(\varepsilon^*+\varepsilon_0)/2,\quad \varepsilon^*=0{,}38\varepsilon_0; \tag{6}$$

$$\sigma_i(\varepsilon,\varepsilon_i)=a_3(\varepsilon,\varepsilon_i^*)(\varepsilon_i-\varepsilon_i^*)\quad \text{for}\quad \varepsilon_i^*\leqslant\varepsilon_i\leqslant(\varepsilon_i)_3,$$
$$\sigma_i(\varepsilon,\varepsilon_i)=a_3(\varepsilon,\varepsilon_i^*)[(\varepsilon_i)_3-\varepsilon_i^*]^2+\psi_i(\varepsilon)[\varepsilon_i-(\varepsilon_i)_3]\quad \text{for}\quad (\varepsilon_i)_3<\varepsilon<(\varepsilon_i)_2,$$
$$\sigma_i(\varepsilon,\varepsilon_i)=a_4(\varepsilon,\varepsilon_i^0)\varepsilon_i^2+b_4(\varepsilon_i,\varepsilon_i^0)\varepsilon_i+c_4(\varepsilon_i,\varepsilon_i^0)\quad \text{for}\quad (\varepsilon_i)_2<\varepsilon_i\leqslant\varepsilon_i^0,$$
$$a_4=\frac{5\psi_i(\varepsilon)}{\sigma_i(\varepsilon,\varepsilon_i^0)}\left[\frac{\partial}{\partial\varepsilon_i}\sigma_i(\varepsilon,\varepsilon_i^0)-\psi_i(\varepsilon)\right],\quad b_4=\frac{\partial}{\partial\varepsilon_i}\sigma_i(\varepsilon,\varepsilon_i^0)-2a_4\varepsilon_i^0,$$
$$c_4=\sigma_i(\varepsilon,\varepsilon_i^0)-a_4(\varepsilon_i^0)^2-b_4\varepsilon_i^0, \tag{7}$$
$$(\varepsilon_i)_2=\varepsilon_i^0-\frac{\sigma_i(\varepsilon,\varepsilon_i^0)}{10\,\psi_i(\varepsilon)},\quad \bar{\sigma}_i=\frac{\sigma_i(\varepsilon,\varepsilon_i^0)}{10},$$
$$(\varepsilon_i)_3=(\varepsilon_i)_2+\frac{\bar{\sigma}_i-[a_4(\varepsilon_i)_2^2+b_4(\varepsilon_i)_2+c_4]}{\psi_i(\varepsilon)},$$
$$\varepsilon_i^*=(\varepsilon_i)_3-\frac{2\bar{\sigma}_i}{\psi_i(\varepsilon)},\quad a_3=\frac{1}{2}\psi_i(\varepsilon)/[(\varepsilon_i)_3-\varepsilon_i^*],$$
$$\psi_i(\varepsilon)=2303{,}8+80[\sigma(\varepsilon)-25].$$

Here ε_0, ε_i^0 are the values of bulk deformation and the deformation intensity at the loading branches at the start of unloading, the functions $\sigma(\varepsilon_0)$, $\sigma'(\varepsilon_0)$ and $\sigma_i(\varepsilon,\varepsilon_i^0)$, $\frac{\partial}{\partial\varepsilon_i}\sigma_i(\varepsilon,\varepsilon_i^0)$ are calculated, respectively, on the basis of Eqs. (4) and (5), which are valid for loading of the medium, ε^*, ε_i^* are the residual deformation and the deformation intensity, $(\varepsilon_i)_2$, $(\varepsilon_i)_3$ are the deformation intensity values at the edges of the intermediate portion of the separation loading branch of the diagram $\sigma_i=\sigma_i(\varepsilon,\varepsilon_i)$; and the prime denotes differentiation with respect to the argument. In Eqs. (6) and (7) the functions σ_i, σ, $\psi_i(\varepsilon)$ have the dimension of kilogram per centimeter squared.

As external layer of the ring-shaped element of the cylinder we took various porous materials, for which the experimental dependence $\sigma=\sigma(\varepsilon)$ and the Poisson coefficient are known. These parameters make it possible to determine the generalized Lamé coefficients λ and G appearing in (2) for loading and unloading of the external layer material of the cylinder. Thus, for foam epoxy with $\rho_{II}=20$ kg·sec²/m⁴ and $\nu=0.1$ the experimental curve $\sigma=\sigma(\varepsilon)$ was approximated in the form of a second order polynomial with account of residual deformation:

For loading

$$\sigma(\varepsilon)=441\varepsilon-540\varepsilon^2\quad \text{for}\quad 0\leqslant\varepsilon\leqslant0{,}4,$$
$$\sigma(\varepsilon)=86{,}4-9\varepsilon\quad \text{for}\quad \varepsilon>0{,}4; \tag{8}$$

for unloading

$$\sigma(\varepsilon)=a_5(\varepsilon-\varepsilon^*)^2\quad \text{for}\quad \varepsilon^*\leqslant\varepsilon\leqslant\varepsilon_3,$$
$$\sigma(\varepsilon)=a_5(\varepsilon_3-\varepsilon^*)^2+\psi_0(\varepsilon-\varepsilon_3)\quad \text{for}\quad \varepsilon_3<\varepsilon\leqslant\varepsilon_2,$$
$$\sigma(\varepsilon)=a_6\varepsilon^2+b_6\varepsilon+c_6\quad \text{for}\quad \varepsilon_2<\varepsilon\leqslant\varepsilon_0,$$
$$a_6=\frac{5\psi_0}{\sigma(\varepsilon_0)}[\sigma'(\varepsilon_0)-\psi_0],\quad b_6=\sigma'(\varepsilon_0)-2a_6\varepsilon_0, \tag{9}$$
$$c_6=\sigma(\varepsilon_0)-a_6\varepsilon_0^2-b_6\varepsilon_0,\quad \varepsilon_2=\varepsilon_0-\sigma(\varepsilon_0)/(10\psi_0'),\quad \bar{\sigma}=\sigma(\varepsilon_0)/10,$$
$$\varepsilon_3=\varepsilon_2+[\bar{\sigma}-(a_6\varepsilon_2^2+b_6\varepsilon_2+c_6)]/\psi_0,\quad \varepsilon^*=\varepsilon_3-2\bar{\sigma}/\psi_0,$$
$$a_5=\psi_0/[2(\varepsilon_3-\varepsilon^*)],\ \psi_0=500\ \text{kg/cm}^2$$

(ε_2, ε_3 are the deformation values at the edges of the intermediate portion of the separation branch of the unloading diagram $\sigma=\sigma(\varepsilon)$, and the functions $\sigma(\varepsilon_0)$, $\sigma'(\varepsilon_0)$ are found from Eq. (8) for $\varepsilon=\varepsilon_0$).

The internal layer of the cylinder was filled by reinforced concrete, which is assumed to be an elastic medium with $\rho_b=250$ kg·sec²/m⁴, $\nu=0.25$ and a Young modulus of $E=3\cdot10^5$

kg/cm^2. In the general case the method takes into account the elastoplastic properties of the internal layer of the cylinder within deformation theory [1, 2].

The initial conditions of the problem are the ground parameters behind the front of the leading wave at the cylinder, which are assumed to be given functions of time. As boundary conditions we take the continuity conditions of the displacement, as well as of the normal and tangential stresses at the contact surfaces ($r = R_i$, $i = 1, 2$) between the media, and absence of stresses on the internal surface ($r = r_0$) of the cylinder.

To solve the problem we use the finite difference method [4]. For this purpose the investigated three-layered region with a conditional external boundary $R_3 \approx (5\text{-}10)R_2$, being a rectangle $r_0 \leq r \leq R_3$, $0 \leq \varphi \leq \pi$, is covered by a grid with radial Δr_1, Δr_2, Δr_3 and polar angle $\Delta\varphi$ sizes including the satisfaction of stability conditions of the difference scheme $(a_0\Delta t)/\Delta r < 1/2$ (a_0 is the rate of propation of the elastic wave). Similarly to the method of [9], we assume that near the boundary $r = R_3$ and in the region of absence of wave reflection from the cylinder the ground motion is determined by the plane wave parameters of given intensity $p(t)$. At the front of this wave relative to the horizontal direction Ox, coinciding with the direction of Or at $\varphi = 0$, we have

$$\rho(D - u_t) = \rho_0 D, \quad \sigma_{xx} = -\rho_0 D u_t \tag{10}$$

(D, ρ, u_t are the front velocity of the shock wave, the density, and the mass velocity of the ground).

Using for the stress σ_{xx} a dependence of the type (2) with account of (5), we find

$$\sigma_{xx} = \Big\{\sigma(\varepsilon) - \frac{2}{3}\Big[\sigma_i^{н}(\varepsilon_i) - \frac{1}{15}\big(\sigma_i^{в}(\varepsilon_i) - \sigma_i^{н}(\varepsilon_i)\big) \times \times (\sigma(\varepsilon) + 25)\Big]\Big\}\Big(1 + \frac{\partial u}{\partial x}\Big). \tag{11}$$

Here $\varepsilon = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2$; $\varepsilon_i = -\frac{2}{3}\varepsilon$; $\varepsilon < 0$; and u is the ground displacement in the Ox direction. from approximate graphical analysis of (11) and approximation of the corresponding curve for $|\sigma_{xx}| \leq 200$ kg/cm^2, $|\partial u/\partial x| \leq 0.06$ with account of the condition $|\sigma_{xx}| = p(t)$ for $x = s(t) \approx R_2 - at$, where $a = D$ at $t = 0$, we obtain

$$u_x(x, t) = \frac{\partial u}{\partial x}(x, t) = -\frac{p\left(\frac{x - R_2}{a} + t\right)}{2{,}8\cdot 10^7}\left[9210 - 0{,}00135p\left(\frac{x - R_2}{a} + t\right)\right]. \tag{12}$$

To determine the front velocity of the plane wave we have for $t = 0$ from (10) with account of $\rho = \rho_0/(1 + u_x)$

$$a = D(0) = \sqrt{\frac{p_0}{\rho_0 |u_x(0)|}} \quad \text{for } |u_x(0)| = \frac{p(0)}{2{,}8\cdot 10^7}[9210 - 0{,}00135p(0)].$$

The ground displacement $u(x, t)$ and velocity $\dot{u}(x, t)$ behind the front of the planar shock wave in the region of absence of body effects are then represented, with account of (12), in the form

$$u(x, t) = -\frac{1}{2{,}8\cdot 10^7}\int_{s(t)}^{x} p\left(\frac{x - R_2}{a} + t\right)\Big[9210 - 0{,}00135p\left(\frac{x - R_2}{a} + t\right)\Big]dx; \tag{13}$$

$$\dot{u}(x, t) = -\frac{1}{2{,}8\cdot 10^7}\int_{s(t)}^{x} p'\left(\frac{x - R_2}{a} + t\right)\Big[9210 - 0{,}00270p\left(\frac{x - R_2}{a} + t\right)\Big]dx + \frac{s'(t)}{2{,}8\cdot 10^7}\, p\left(\frac{s(t) - R_2}{a} + t\right)\Big[9210 - 0{,}00135p\left(\frac{s(t) - R_2}{a} + t\right)\Big], \tag{14}$$

where the upper prime denotes differentiation with respect to time. Transition to a polar coordinate system (r, φ) with account of (13) and (14) gives

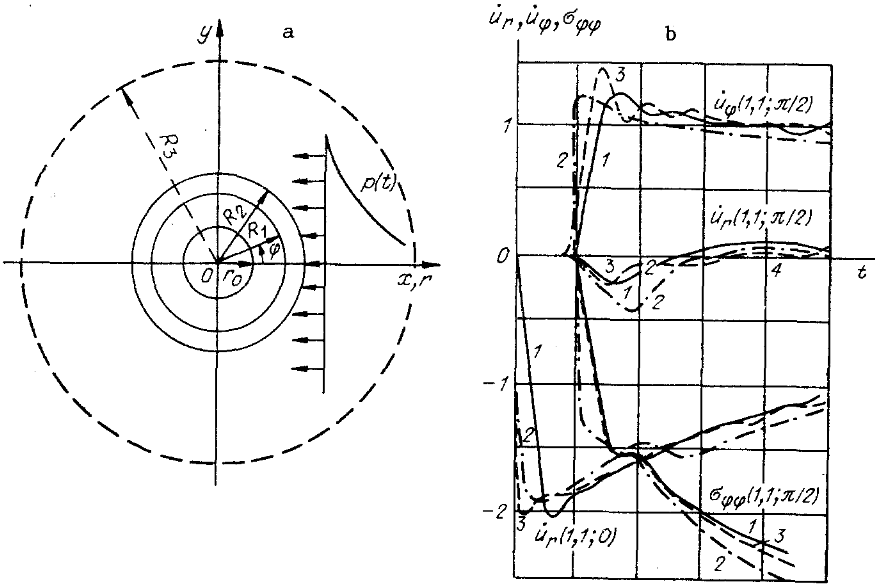

Fig. 1

$$u_r(r,\ \varphi,\ t)=u(r\cos\varphi,\ t),\ u_\varphi(r,\ \varphi,\ t)=-u(r\cos\varphi,\ t)\sin\varphi, \qquad (15)$$
$$\dot{u}_r(r,\ \varphi,\ t)=\dot{u}(r\cos\varphi,\ t)\cos\varphi,\ \dot{u}_\varphi(r,\ \varphi,\ t)=-\dot{u}(r\cos\varphi,\ t)\sin\varphi.$$

Thus, at the external conditional boundary $r = R_3$ of the region considered we obtain: for $x_m = R_3\cos\varphi \geqslant s(t^{(k)})$

$$u_r(R_3,\ \varphi,\ t)=u(R_3\cos\varphi,\ t)\cos\varphi,$$
$$u_\varphi(R_3,\ \varphi,\ t)=-u(R_3\cos\varphi,\ t)\sin\varphi,$$
$$\dot{u}_r(R_3,\ \varphi,\ t)=\dot{u}(R_3\cos\varphi,\ t)\cos\varphi,$$
$$\dot{u}_\varphi(R_3,\ \varphi,\ t)=-\dot{u}(R_3\cos\varphi,\ t)\sin\varphi;$$

for $x_m < s(t^{(k)})$

$$u_r(R_3,\ \varphi,\ t)=u_\varphi(R_3,\ \varphi,\ t)=\dot{u}_r(R_3,\ \varphi,\ t)=$$
$$=\dot{u}_\varphi(R_3,\ \varphi,\ t)=0.$$

The initial conditions of the problem for $t \leq 0$ are the commonly adopted equations for the displacements, velocities (15), and stresses (2) with account of (13), (14), and (3)-(5), depending directly on the loading profile $p(\xi)(\xi = (s(t) - R_2 + at)/a)$ at the front of the plane wave. Since $t = 0$ is taken to be the time at which the planar shock wave has reached the surface of the cylinder at the point $s(t) = x = R_2$ (Fig. 1a), from (15) we find with account of (13) and (14) at $t = 0$

$$u_r(r,\ \varphi,\ 0)=u_\varphi(r,\ \varphi,\ 0)=0,$$

$$\dot{u}_r(r,\ \varphi,\ 0)=-\frac{a}{2{,}8\cdot 10^7}p(0)\,[9210-0{,}00135p(0)]\cos\varphi,$$
$$\dot{u}_\varphi(r,\ \varphi,\ 0)=\frac{a}{2{,}8\cdot 10^7}p(0)\,[9210-0{,}00135p(0)]\sin\varphi,$$

where $r = R_2/\cos\varphi$. Differentiating, further, the first two Eqs. of (15) with respect to r and φ, and substituting them into (3), we have expressions for ε_{rr}, $\varepsilon_{\varphi\varphi}$, $\varepsilon_{r\varphi}$ and ε, ε_i at $t = 0$, with whose use and that of the dependences (4), (5) one obtains from (2) at $t = 0$ equations for σ_{rr}, $\sigma_{\varphi\varphi}$, $\sigma_{r\varphi}$ as functions of r and φ (they are not given due to their unwieldiness).

Due to the symmetry with respect to the plane $\varphi = 0$ the solution of the problem is determined for angles $0 \leqslant \varphi \leqslant \pi$, while for $\varphi = 0$ and $\varphi = \pi$ one has the conditions

$$\sigma_{r\varphi}=\dot{u}_\varphi=0,\quad \frac{\partial u_r}{\partial\varphi}=\frac{\partial\sigma_{rr}}{\partial\varphi}=\frac{\partial\sigma_{\varphi\varphi}}{\partial\varphi}=0.$$

To calculate values of partial derivatives of the functions considered in (1) with respect to r and φ in interior points of the region one uses a difference scheme of the "spatial cross" type with second order accuracy approximations. In this case it is assumed that at the moments of time $t = k\Delta t$ (k is a positive integer) one knows the displacements u_r, u_φ, including the stress components, while at the moments of time $t = k - 1/2)\Delta t$ one knows the velocities $\dot{u}_r, \dot{u}_\varphi$. From the known u_r, u_φ one calculates the first part of system (1) and one finds the acceleration values $\ddot{u}_r, \ddot{u}_\varphi$ for the points indicated at $t = k\Delta t$. Following the calculation of the acceleration values one determined the velocities $t = (k + 1/2)\Delta t$, at the moments of time $t = (k + 1/2)\Delta t$, and then the displacements u_r, u_φ at $t = (k + 1)\Delta t$. The calculation is concluded at this next cycle.

To "smear" the jumps and "quench" the oscillations one stipulates pseudostresses with a synthetic viscosity in the form

$$\sigma_{rr}^{\text{в}} = \sigma_{\varphi\varphi}^{\text{в}} = k_1 \left(\frac{\partial \varepsilon}{\partial t}\right)^2, \quad \sigma_{r\varphi}^{\text{в}} = k_2 \left(\frac{\partial \varepsilon_i}{\partial t}\right)^2. \tag{16}$$

Here the coefficients k_1, k_2 are determined directly by a numerical experiment.

A test of the numerical scheme developed was carried out on the test case of interaction of a nonstationary plane wave with a cylindrical plane in an elastic medium [9, 10]. In the calculations the loading behind the front of the wave streamlining the cylinder is given in the form of steps, "smeared" into two phases in time. The calculation results are shown in Fig. 1b, where 1 and 2 are curves referring to [9] and [10], and 3 are results obtained by the given method. Comparison of these results shows that the results of the method suggested and those of earlier studies by other authors are in quite satisfactory agreement. Consequently, the use of the suggested difference method to describe two-dimensional wave processes is valid.

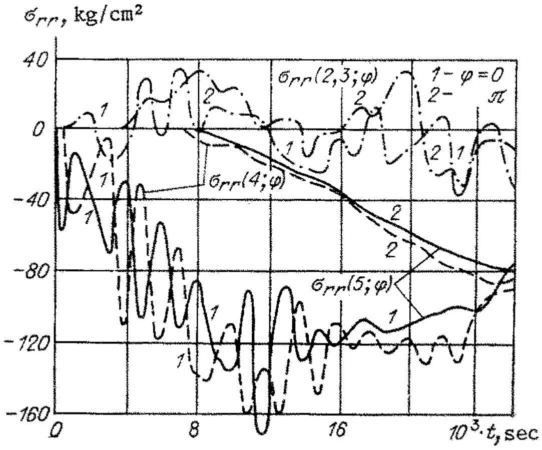

Fig.2

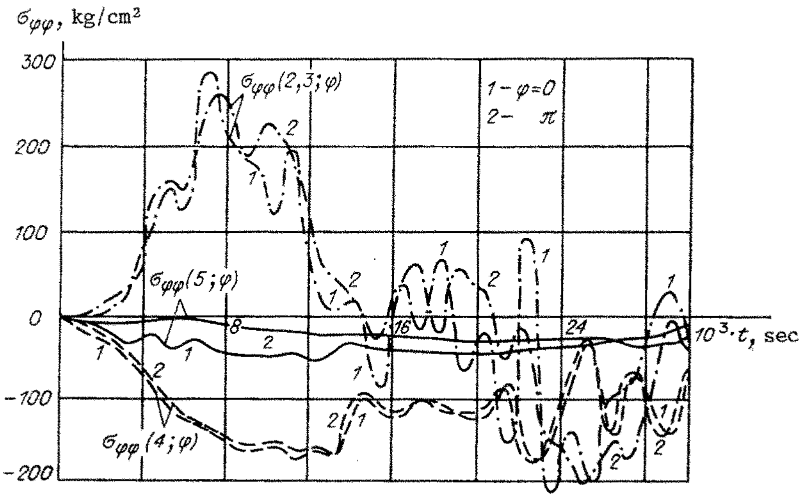

Fig. 3

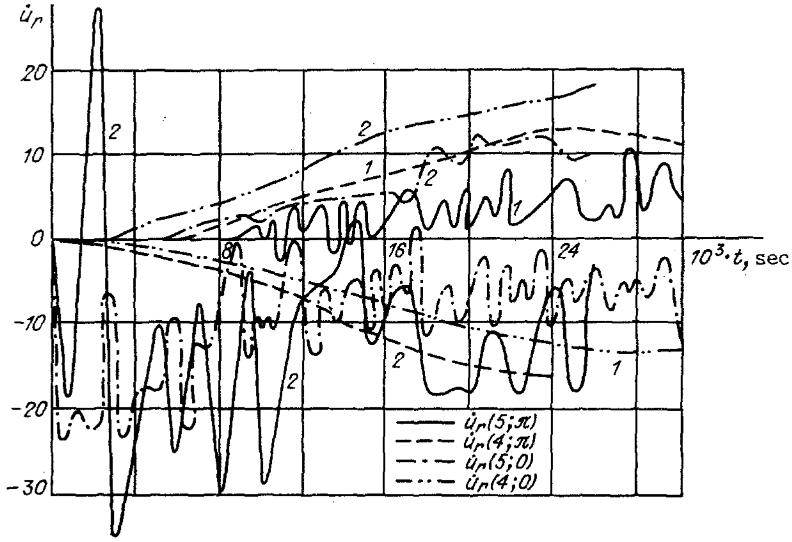

Fig. 4

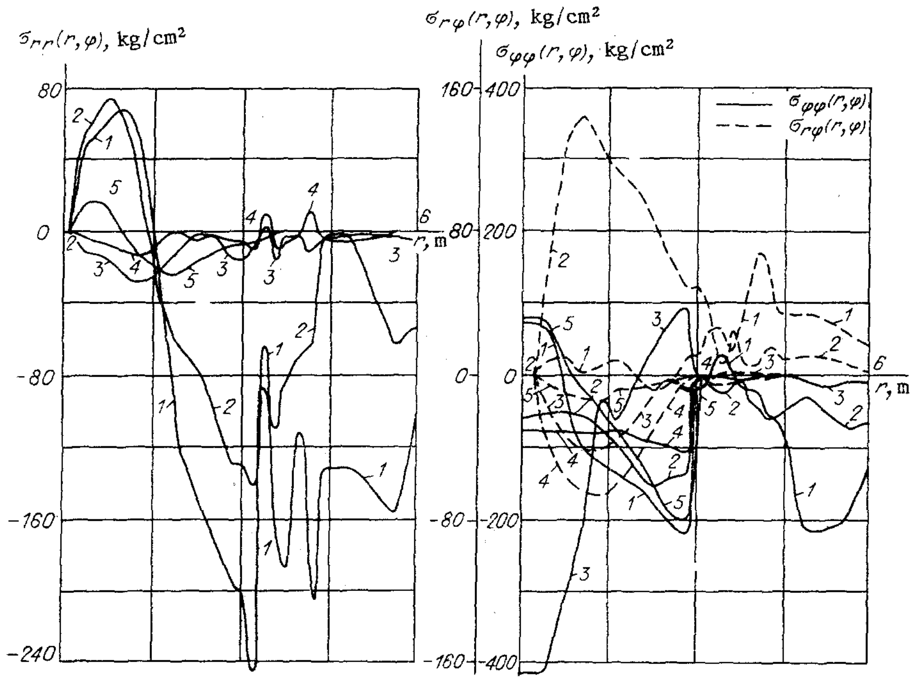

Fig. 5

Computer calculations have been further carried out for the case in which the given pressure p(t) in the ground behind the front of the plane wave varies by the law

$$p(t) = \frac{p(0)}{0{,}018}[(t-0{,}03)(t-0{,}06)q_0 - 2t(t-0{,}06)q_1 + t(t-0{,}03)q_2],$$
$$p(0) = 100 \text{ kg/cm}^2,\ q_0 = 1,\ q_1 = 0{,}18,\ q_2 = 0{,}1, \quad (17)$$

where the geometric sizes of the cylinder and of the ground are: r_0 = 2 m, R_1 = 4 m, R_2 = 5 m, R_3 = 35 m. In this case we select the following original data for the cells sizes and for the step in time:

$$\Delta r_1 = 0{,}2 \text{ m},\ \Delta r_2 = 0{,}1 \text{ m},$$
$$\Delta r_3 = 0{,}5 \text{ m},\quad \Delta\varphi = \pi/30, \quad (18)$$
$$\Delta t = 2 \cdot 10^{-5} \text{ sec}.$$

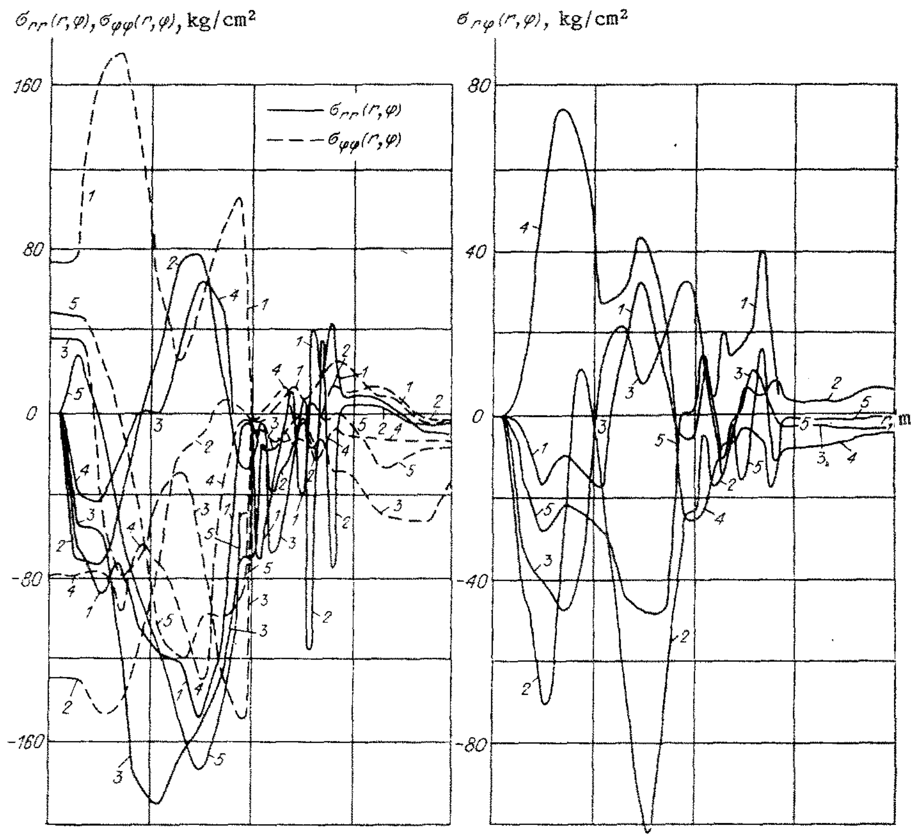

Fig. 6

In (18) the step in time Δt was selected from the condition $\Delta t a_b/\Delta r < 1/2$, where Δr_1 is the step in r in the internal layer of the cylinder, and a_b is the propagation velocity of the elastic longitudinal wave in reinforced concrete.

Several calculation results in the form of stress plots σ_{rr}, $\sigma_{\varphi\varphi}$ and mass velocities u_r are provided in Figs. 2-7; while Figs. 2-6 correspond to elastoplastic deformation of the ground with account of (4)-(7) and of the external layer of the cylinder, consisting of foam epoxy, Fig. 7 corresponds to the linearly elastic three-layer system. Here, however, on the basis of handling results of a series of numerical experiments it must be stressed that the synthetic viscosity (16) and further reduction of the coordinate grid do not affect substantially the stress distribution, and therefore in all calculations the coefficients k_1 and k_2 in (16) were taken to be zero.

Analysis of the curves of Figs. 2, 3, and 7 shows that the stress σ_{rr} in the cross sections r = 5, 4, 2.3 acquires the largest value in the elastoplastic case in comparison with the elastic case. This is related to the fact that in bulk compression the ground displays a shock diagram, leading to an enhanced stress.

In the leading part of the body, i.e., for $\varphi = 0$, the stress distribution σ_{rr} as a function of time t does not differ substantially at r = 5 and 4 m and is compressive. In the cross sections r = 2, 3 m inside the reinforced concrete of the ring-shaped element of the cylinder σ_{rr} is obtained by time alternating and somewhat decreased values than at the boundary r = 4 m between the cylinder layers.

In the rear part of the cylinder, i.e., at $\varphi = \pi$ with account of perturbation lag and some attenuation in wave intensity the stress amplitude obtained is somewhat smaller than for $\varphi = 0$.

The elastic and elastoplastic annular stress $\sigma_{\varphi\varphi}$ at various points of the cylinder varies substantially and nonlinearly as a function of t for $\varphi = 0, \pi$ (Figs. 3, 7), while for r = 2, 3 m the elastoplastic stress $\sigma_{\varphi\varphi}$ changes sign and has the largest value in comparison with the corresponding elastic stress.

We note that in all cases the behavior of the curves σ_{rr} and $\sigma_{\varphi\varphi}$ has an oscillatory nature as a function of time (Figs. 2, 3, 7). This is due to the presence of different

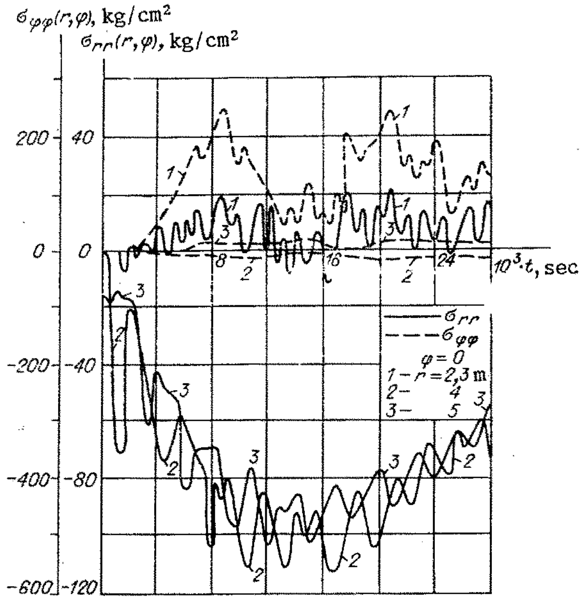

Fig. 7

cylinder materials and wave diffraction processes by the media contact boundaries. Thus, the investigation of the dynamic behavior of the elastic stress σ_{rr} for r = 5 m, $\varphi = 0$ (Fig. 7) shows that at the initial phase of wave interaction with a two-layered cylinder, at $t \leq 10^{-2}$ sec and following a half-period oscillation of the curve σ_{rr} approximately equal to the transmission time of the elastic longitudinal wave through the two widths of the external layers of the cylinder, consisting of foam epoxy, the propagation velocity of the elastic longitudinal wave in the foam epoxy is $a_b \approx 1000$ m/sec.

Besides, for a homogeneous cylinder the oscillations mentioned above do not lead to other effects, and the maximum σ_{rr} value at the cylinder is somewhat enhanced.

Investigating the curve behavior for the velocity $\dot{u}_r$ for elastic (Fig. 4, curves 1) and elastoplastic (curves 2) medium deformation, we note that $\dot{u}_r$ reaches its maximum value when irreversible processes are taken into account. In this case, depending on t the $\dot{u}_r$ curve has at r = 5 m, $\varphi = 0$ a change in sign and an attenuation in absolute value, while at r = 4 m, $\varphi = 0$, π first increases, then decreases.

Analyzing the nature of the distribution of the elastoplastic stress σ_{rr} over the angle φ, we note that at the moment of time t = 0.012 (0.020) sec, when the front of the planar shock wave streamlining around the cylinder covers a distance equal to x = 5.5 (9) m, for r = 5 and 4 m and in the interval $0 \leqslant \varphi \leqslant \pi/2$ it varies by the law of damped oscillations, while in the interval $\pi/2 \leqslant \varphi \leqslant \pi$ one observes in the σ_{rr} profile a tendency to increase. While the curves for σ_{rr}, $\sigma_{\varphi\varphi}$, $\sigma_{r\varphi}$ at r = 2, 3 m, as well as the curves for the tangential stress $\sigma_{r\varphi}$ at r = 5 and 4 m are alternating as a function of φ are nonsymmetric with respect to the point $\varphi = \pi/2$. Consequently, the distribution of the normal and the tangential stresses at the external surface of the cylinder has a nonuniform nature as a function of the polar angle φ.

Since one solves the two-dimensional nonstationary problem, for prognosis of the possible nature of breakdown of the cylinder material and to estimate the expansion stresses it is advisable to study the stress distribution in the bulk of a two-layered cylinder and around it at fixed moments of time. In this connection we show in Figs. 5, 6, the distribution curves of σ_{rr}, $\sigma_{\varphi\varphi}$ and $\sigma_{r\varphi}$ over r in the range 2 m < r < 6 m for φ = 3, 45, 90, 135, 177° (curves 1-5) at the moments of time t = 0.0108 and 0.0216 sec (Figs. 5 and 6), when the plane wave traverses, respectively, the radius and diameter of the cylinder, and acts on it with a load decreasing monotonically with time (17). It is hence seen that the stress components σ_{rr}, $\sigma_{\varphi\varphi}$, $\sigma_{r\varphi}$ are basically alternating in a two-layered cylinder as a function of r. In the case of t = 0.0108 sec, near the internal surface of the cylinder at 2 m $\leq$ r < 3 m, and for small φ values and φ = 177° (curves 1, 2, 5) there occur regions of stretching stresses σ_{rr}, $\sigma_{r\varphi}$, $\sigma_{\varphi\varphi}$. At the moment of time t = 0.0216 sec, when the wave traverses the diameter of the cylinder, for 2 m $\leq$ r < 3 m and $\varphi = 3°$ (Fig. 6, curve 1) the positive amplitude of the circular stress $\sigma_{\varphi\varphi}$ is somewhat larger than at t = 0.0108 sec, and the stretching zone stresses σ_{rr} at φ = 45 and 135° (Fig. 6, curves 2, 4) are displaced inside ($r \geq 3$ m) the cylindrical layer. Near the cylinder, at r > 5 m the stress as a function of r is substantially reduced in the time interval under consideration.

Further investigations and analysis of the results show that fittings of reinforced concrete structures outside a protective padding of given thickness of a more compliant material make it possible to reduce the dynamic load amplitudes at the cylinder surface under the action of an intense seismic detonation wave. At the same time the level of load reduction at the annular element of a thick-walled cylinder depends substantially on the physicomechanical characteristics of the padding material and on its thickness.

Thus, the variations in kinematic parameters and stress distributions at various fixed points of the cylinder as a function of time and spatial coordinates possess as a whole complex nonlinear wave properties, and for the prognosis of stress-deformation states of a two-layered cylinder under the action of an intense compressional wave it is necessary to take into account inelastic irreversible processes, generated both in the ground and in the padding, making it possible to refine the results of the corresponding elastic problem of wave interaction with a rigid or deformable homogeneous cylinder in the ground.

We note that a similar study has been carried out of the stress-deformation state of a three-layered cylinder under the action of an intense seismic detonation wave on it.

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